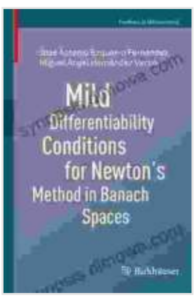
Mild Differentiability Conditions for Newton Method in Banach Spaces

The Newton method is a powerful tool for solving nonlinear equations. It is based on the idea of iteratively linearizing the equation around the current estimate of the solution. This linearization is typically done using the derivative of the function. However, in many applications, the derivative is not well-defined or is not available. In these cases, it is necessary to relax the differentiability assumptions.



Mild Differentiability Conditions for Newton's Method in Banach Spaces (Frontiers in Mathematics)

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4.6 out of 5

Language	: English
File size	: 6085 KB
Screen Reader	: Supported
Print length	: 192 pages

This article provides a comprehensive overview of mild differentiability conditions for the Newton method in Banach spaces. Mild differentiability conditions are weaker than the usual differentiability conditions and allow the Newton method to be used in a wider range of applications.

Mild Differentiability Conditions

A function $f: X \to Y$ is said to be mildly differentiable at $x \in X$ if there exists a bounded linear operator $L: X \to Y$ such that

$$\lim_{h \to 0}\frac{f(x+h) - f(x) - Lh}{\|h\|}= 0.$$

The operator L is called the mild derivative of f at x.

Mild differentiability conditions are weaker than the usual differentiability conditions. In particular, a function that is mildly differentiable at x is not necessarily differentiable at x. However, mild differentiability conditions are sufficient to guarantee the convergence of the Newton method.

Convergence of the Newton Method

The Newton method is given by the following iteration:

$$x_{n+1}= x_n - f'(x_n)^{-1}f(x_n),$$

where $f': X \to L(X, Y)$ is the derivative of f. If f is mildly differentiable at x and x_0 is sufficiently close to x, then the Newton method will converge to x.

The rate of convergence of the Newton method depends on the mild derivative of f. If the mild derivative is Lipschitz continuous, then the Newton method will converge quadratically. If the mild derivative is only Hölder continuous, then the Newton method will converge linearly.

Applications

The Newton method with mild differentiability conditions has been used in a wide range of applications, including:

* Solving nonlinear equations in Banach spaces * Finding roots of polynomials * Solving optimization problems * Solving integral equations *

Solving partial differential equations

Mild differentiability conditions are a powerful tool for extending the applicability of the Newton method. They allow the Newton method to be used to solve a wider range of nonlinear equations, even when the derivative of the function is not well-defined or is not available.

References

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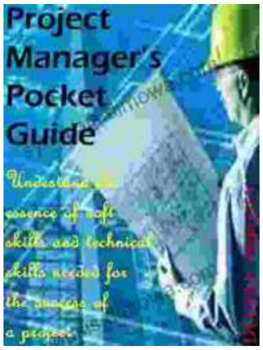

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