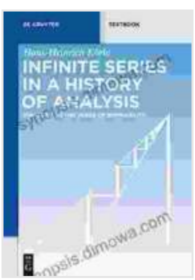
# Stages Up To The Verge Of Summability: A Comprehensive Exploration

The concept of summability plays a pivotal role in various branches of mathematics, including analysis, harmonic analysis, and differential equations. Its importance lies in providing a framework for understanding the convergence and divergence of infinite series, which has wide-ranging applications in fields such as numerical analysis, probability theory, and control theory. In this context, the book "Stages Up To The Verge Of Summability" by De Gruyter Textbook offers a comprehensive and in-depth treatment of this fundamental mathematical concept.




**Infinite Series in a History of Analysis: Stages up to the Verge of Summability (De Gruyter Textbook)** by David Sinclair

4.1 out of 5

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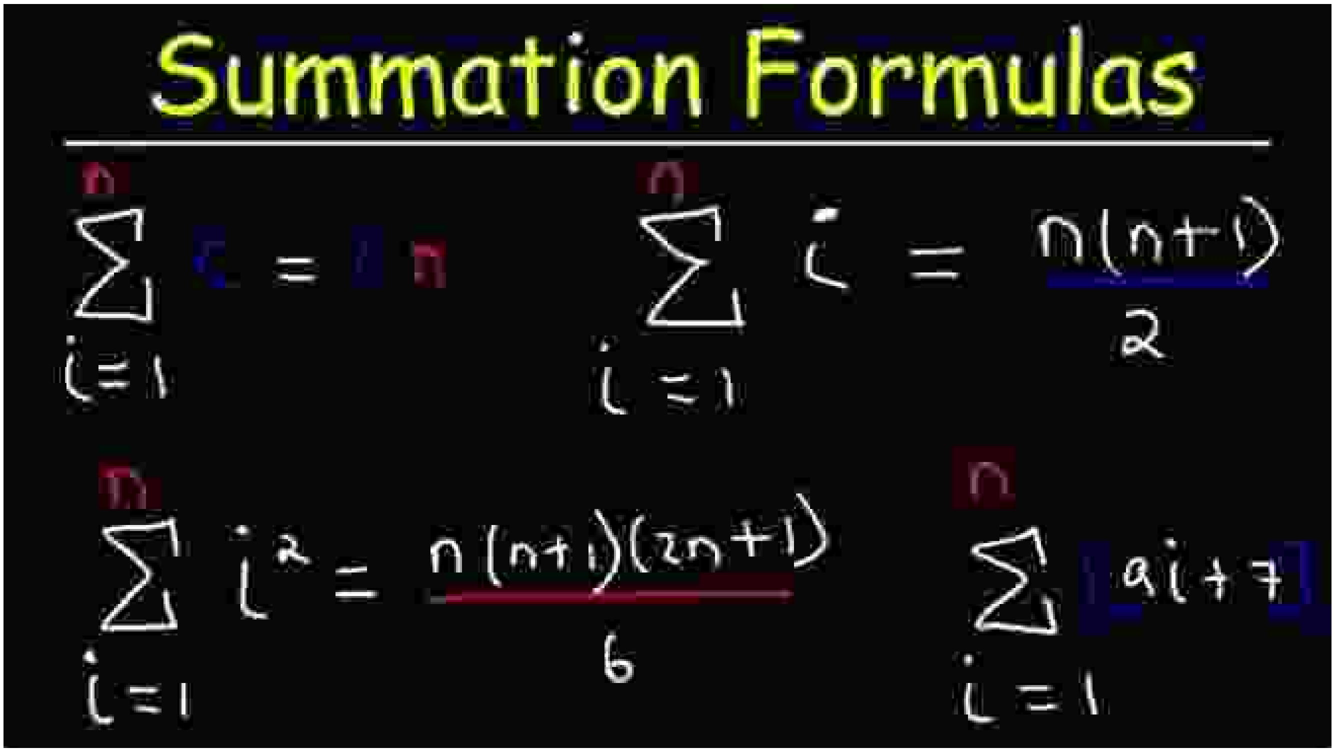
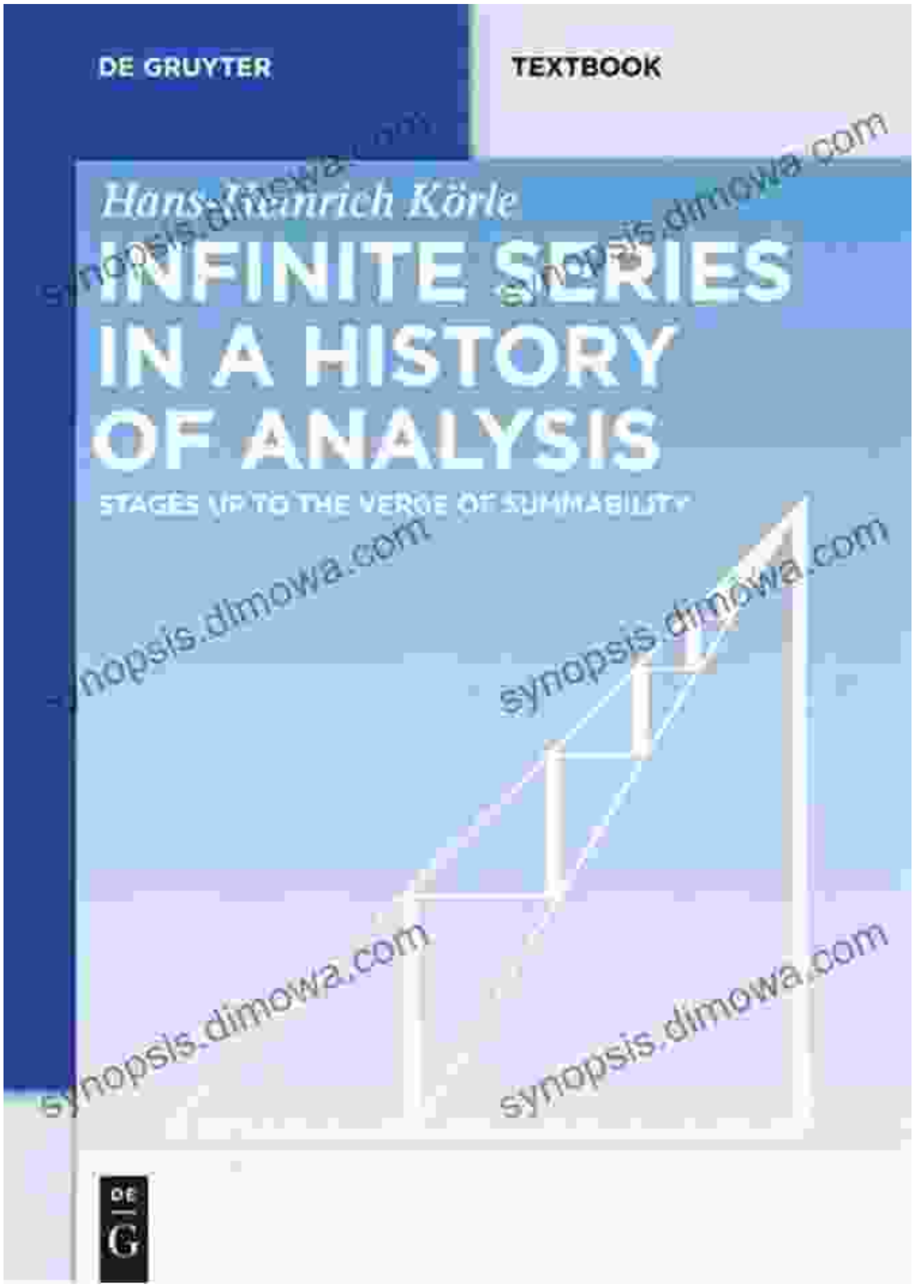
## Chapter 1: The Basics of Summability

This chapter provides an to the fundamental concepts related to summability. It begins with a detailed discussion of convergence and divergence tests for series, including the Cauchy criterion, the ratio test, and the root test. The chapter then introduces various summability methods, such as the Cesàro method, the Abel method, and the Borel method. These methods provide alternative ways to assign values to divergent series, thereby extending the concept of convergence.




## Chapter 2: Strong Summability

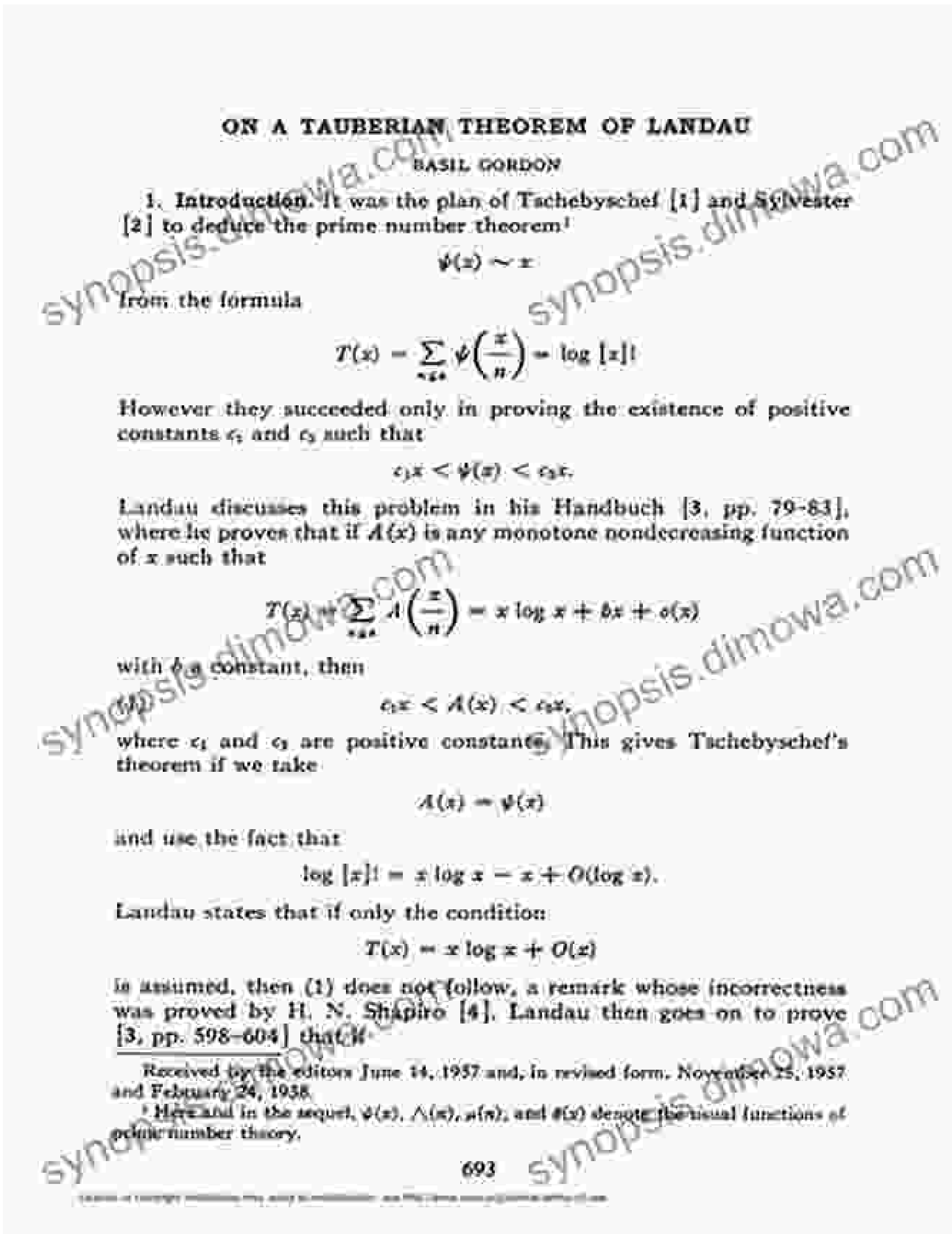
Chapter 2 delves into the concept of strong summability, which is a more restrictive notion than ordinary summability. Strong summability methods require the convergence of the series to be uniform in some sense. The chapter explores various types of strong summability, including absolute summability, strong Cesàro summability, and strong Borel summability. These methods are particularly useful in studying the convergence of Fourier series and other orthogonal expansions.




Exploring the intricacies of strong summability

## Chapter 3: Tauberian Theorems

Chapter 3 introduces Tauberian theorems, which provide connections between the summability of a series and the asymptotic behavior of its terms. These theorems play a crucial role in analyzing the asymptotic behavior of solutions to differential equations and integral equations. The chapter covers classical Tauberian theorems, such as the Hardy-Littlewood theorem and the Wiener-Tauberian theorem, as well as their applications in various areas of mathematics.



ON A TAUBERIAN THEOREM OF LANDAU

BASIL GORDON

1. **Introduction.** It was the plan of Tschebyschef [1] and Sylvester [2] to deduce the prime number theorem[1]

$$\psi(x) \sim x$$

from the formula

$$T(x) = \sum_{n \le x} \psi\left(\frac{x}{n}\right) = \log [x]!$$

However they succeeded only in proving the existence of positive constants $c_1$ and $c_2$ such that

$$c_1 x < \psi(x) < c_2 x.$$

Landau discusses this problem in his Handbuch [3, pp. 79–83], where he proves that if $A(x)$ is any monotone nondecreasing function of $x$ such that

$$T(x) = \sum_{n \le x} A\left(\frac{x}{n}\right) = x \log x + bx + o(x)$$

with $b$ a constant, then

$$(1) \qquad c_1 x < A(x) < c_2 x,$$

where $c_1$ and $c_2$ are positive constants. This gives Tschebyschef's theorem if we take

$$A(x) = \psi(x)$$

and use the fact that

$$\log [x]! = x \log x - x + O(\log x).$$

Landau states that if only the condition

$$T(x) = x \log x + O(x)$$

is assumed, then (1) does not follow, a remark whose incorrectness was proved by H. N. Shapiro [4]. Landau then goes on to prove [3, pp. 598–604] that if

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[1] Here and in the sequel, $\psi(x)$, $\Lambda(n)$, $\mu(n)$, and $\theta(x)$ denote the usual functions of prime number theory.



## Chapter 4: Summability of Fourier Series

Chapter 4 focuses on the summability of Fourier series, which are essential tools in harmonic analysis and other areas of mathematics. It investigates the convergence and divergence of Fourier series using various summability methods. The chapter also explores the relationship between the summability of a Fourier series and the properties of its coefficients. These results have important applications in studying the smoothness and regularity of functions.



CHAPTER III

## SUMMABILITY OF FOURIER SERIES

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### 1. Summability of numerical series

We consider a doubly infinite matrix

$$M = \begin{pmatrix} a_{00} & a_{01} & \cdots & a_{0\nu} & \cdots \\ a_{10} & a_{11} & \cdots & a_{1\nu} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n0} & a_{n1} & \cdots & a_{n\nu} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

of numbers. With every sequence $s_0, s_1, s_2, \ldots$ we associate the sequence $\{\sigma_n\}$ given by

$$\sigma_n = a_{n0}s_0 + a_{n1}s_1 + \ldots + a_{n\nu}s_\nu + \ldots \quad (n = 0, 1, 2, \ldots), \qquad (1 \cdot 1)$$

provided the series on the right converges for all $n$. If $\sigma_n$ tends to a limit $s$ we shall say that the sequence $\{s_\nu\}$, or the series whose partial sums are $s_\nu$, is *summable* M to limit (sum) $s$. The $\sigma_n$ are also called the *linear means* (determined by the matrix M) of the $\{s_\nu\}$. Matrices M such that $a_{n\nu} = 0$ for $n < \nu$ are called *triangular*.

Let us suppose that the numbers

$$N_n = |a_{n0}| + |a_{n1}| + \ldots, \quad A_n = a_{n0} + a_{n1} + \ldots$$

exist (and are finite) for all $n$. The matrix will be called *regular* if the following conditions are satisfied:

(i) $\lim_{n\to\infty} a_{n\nu} = 0$ for $\nu = 0, 1, \ldots$;

(ii) the $N_n$ are bounded;

(iii) $\lim_{n\to\infty} A_n = 1$.

The finiteness of $N_n$ implies the existence of $A_n$. It also implies the convergence of the series (1·1) for every bounded (in particular, convergent) sequence $\{s_\nu\}$.

(1·2) THEOREM. *If* M *is a regular matrix, and if* $s_\nu$ *tends to a finite limit* $s$, *then* $\sigma_n \to s$.

For let $s_\nu = s + \epsilon_\nu$, $\epsilon_\nu \to 0$. Correspondingly, $\sigma_n = \sigma'_n + \sigma''_n$, where

$$\sigma'_n = sA_n, \quad \sigma''_n = \epsilon_0 a_{n0} + \epsilon_1 a_{n1} + \ldots.$$

Here $\sigma'_n \to s$ by condition (iii). Let $N$ be the upper bound of the $N_n$, and let $|\epsilon_\nu| < \eta/2N$ for $\nu > \nu_0$, where $\eta$ is any given positive number. Then

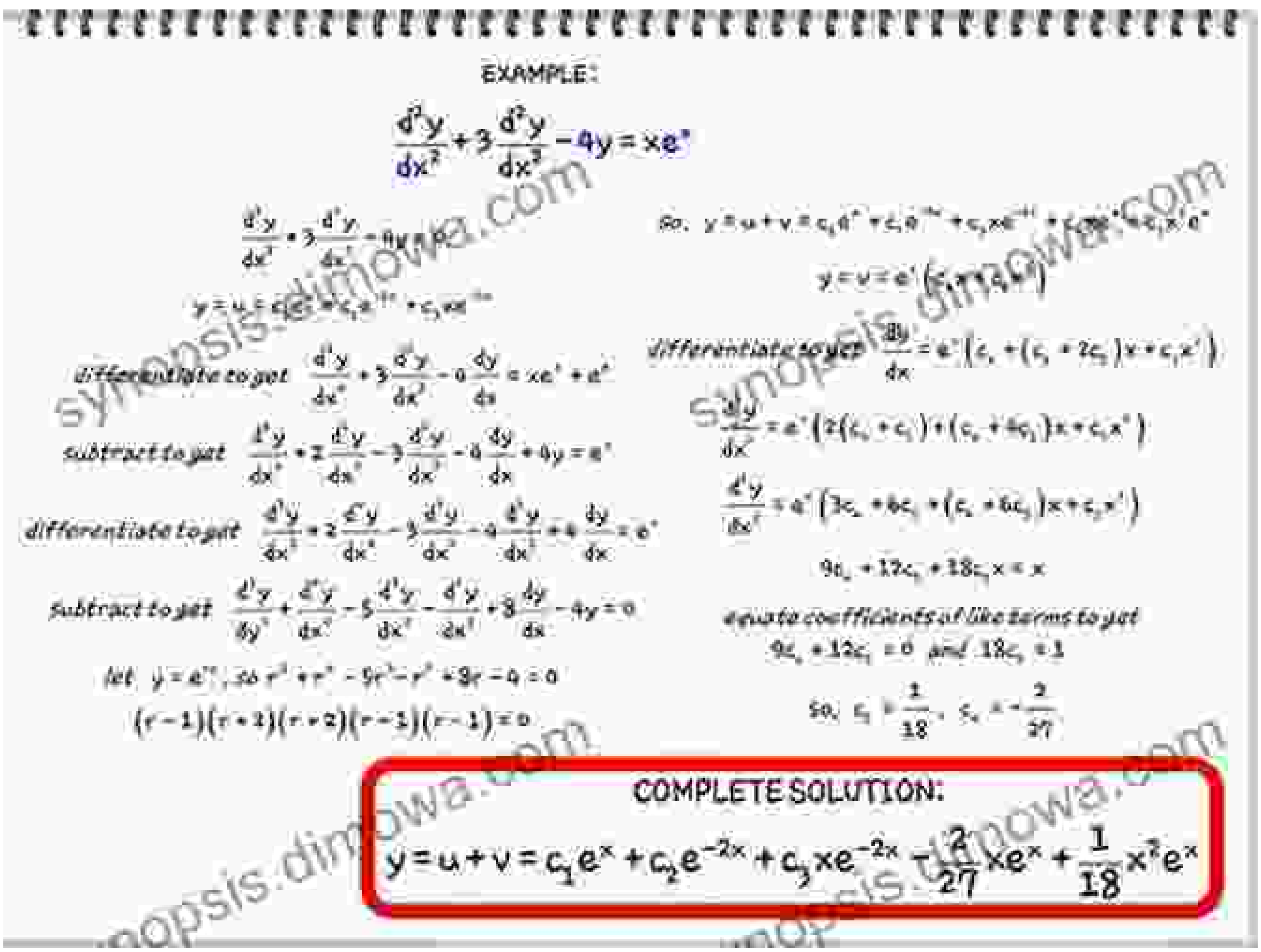
$$|\sigma''_n| \leq (|\epsilon_0||a_{n0}| + \ldots + |\epsilon_{\nu_0}||a_{n\nu_0}|) + (|a_{n,\nu_0+1}| + |a_{n,\nu_0+2}| + \ldots)\eta/2N.$$

The first sum on the right tends to 0 as $n \to \infty$ (condition (i)), and so is less than $\frac{1}{2}\eta$ for $n > n_0$. The remainder does not exceed $N\eta/2N = \frac{1}{2}\eta$. Hence $|\sigma''_n| < \eta$ for $n > n_0$, $\sigma''_n \to 0$, $\sigma_n \to s$, as desired.

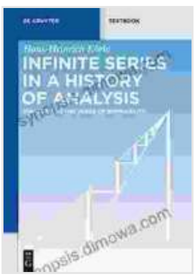
We note that if $s = 0$ condition (iii) is not needed in the above argument.

## Chapter 5: Applications in Differential Equations

Chapter 5 explores the applications of summability in the theory of differential equations. It demonstrates how summability methods can be used to analyze the existence, uniqueness, and asymptotic behavior of solutions to ordinary and partial differential equations. The chapter discusses specific techniques, such as the Laplace transform and the method of steps, which utilize summability to obtain solutions to complex differential equations.



"Stages Up To The Verge Of Summability" by De Gruyter Textbook serves as a comprehensive and authoritative reference for mathematicians, researchers, and students working in areas involving summability. The book's clear and systematic exposition, coupled with its wealth of examples and exercises, makes it an invaluable resource for anyone seeking a deep understanding of this fundamental mathematical concept and its far-reaching applications.

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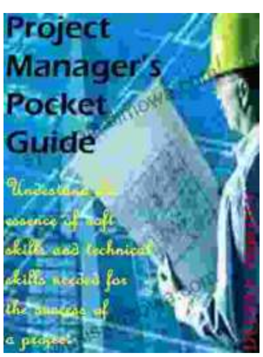


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